Exercice 1 : calculer les dérivées des fonctions suivantes :

a) $f(x) = [x(x-1)(x-2)]^{3/2}$ b) $f(x) = \ln\left|\frac{x(2x-1)}{x+3}\right|$

c) $f(x) = \frac{-2+\ln x}{(\ln x)2}$ d) $f(x) = (x^2-1)\ln(\frac{1+x}{1-x})$

e) $f(x) = e^{\frac{-x2}{2}}$ $|x|^x$ cote f) $f(x) = (\frac{1}{\sqrt{x}})^{\sqrt{x}}$ g) $f(x) = (\sqrt{x})^{x2}$

$$(f^{\alpha})' = \alpha . f' . f^{\alpha - 1}$$

$$f(x) = x(x-1)(x-2) = (x^2 - x)(x-2)$$
$$= x^3 - x^2 - 2x^2 + 2x$$
$$= x^3 - 3x^2 + 2x$$

$$f'(x) = 3x^2 - 6x + 2$$

$$(x^n)' = nx^{n-1}$$

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① $f(x) = \left(x(x-1)(x-2)\right)^{\frac{3}{2}}$

$$\Rightarrow f'(x) = \frac{3}{2} \times \left(x(x-1)(x-2)\right)' . \left(x(x-1)(x-2)\right)^{\frac{3}{2}-1}$$

$$= \frac{3}{2}(3x^2 - 6x + 2)\left(x(x-1)(x-2)\right)^{\frac{1}{2}}$$

$$= \frac{3}{2}(3x^2 - 6x + 2)\sqrt{x(x-1)(x-2)}$$

$$x^{\frac{1}{2}} = \sqrt{x}$$

② $\ln\left|\frac{x(2x-1)}{x+3}\right|$

$$\left[\ln|u(x)|\right]' = \frac{u'(x)}{u(x)}$$

$$u(x) = \frac{x(2x-1)}{x+3} \Rightarrow u'(x) = \frac{(x)'(2x-1) - x(2x-1)'}{(x+3)^2}$$

$$= \frac{1(2x-1) - x(2)}{(x+3)^2}$$

$$= \frac{\cancel{2x} - 1 - \cancel{2x}}{(x+3)^2} = \frac{-1}{(x+3)^2}$$

$$\Rightarrow \quad f'(x) = \frac{\frac{-1}{(x+3)^2}}{\frac{x(2x-1)}{x+3}} = \frac{-1}{(x+3)^{\cancel{2}}} \times \frac{\cancel{x+3}}{x(2x-1)} = \frac{-1}{x(x+3)(2x-1)}$$

(3) $f(x) = \frac{-2+\ln x}{(\ln x)^2}$ $\begin{matrix} u \\ v \end{matrix}$ $\qquad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

Posons $u(x) = -2+\ln x \Rightarrow u'(x) = \frac{1}{x}$

$v(x) = (\ln x)^2 \Rightarrow v'(x) = \frac{2\ln x}{x}$ $\qquad (f^n)' = n f' \times f^{n-1}$

donc $f'(x) = \dfrac{\frac{1}{x}(\ln x)^2 - (-2+\ln x)\cdot\frac{2\ln x}{x}}{(\ln x)^4}$

$$= \frac{\frac{1}{x}(\ln x)^2 + 4\frac{\ln x}{x} - \frac{2(\ln x)^2}{x}}{(\ln x)^4}$$

$$= \frac{\cancel{\ln x}}{x} \times \frac{\ln x + 4 - 2\ln x}{(\ln x)^{\cancel{4}\,3}}$$

$$= \frac{-\ln x + 4}{x(\ln x)^3} = \frac{4-\ln x}{x(\ln x)^3}$$

(4) $f(x) = \underbrace{(x^2-1)}_{u} \times \underbrace{\ln\left(\frac{1+x}{1-x}\right)}_{v}$ $\qquad (u \times v)' = u'v + uv'$

On pose $u(x) = x^2-1 \Rightarrow u'(x) = 2x$

$v(x) = \ln\left(\frac{1+x}{1-x}\right)$

$$\Rightarrow v'(x) = \frac{\left(\frac{1+x}{1-x}\right)'}{\frac{1+x}{1-x}} = \frac{\frac{1\times(1-x)-(1+x)(-1)}{(1-x)^2}}{\frac{1+x}{1-x}} = \frac{1-\cancel{x}+1+\cancel{x}}{(1-x)^{\cancel{2}}} \times \frac{\cancel{(1-x)}}{1+x}$$

$$= \frac{2}{1-x^2}$$

$(1-x)(1+x)$

1^2-x^2

$$f'(x) = 2x \ln\left(\frac{1+x}{1-x}\right) + \overbrace{(x^2-1)}^{-(\cancel{1-x^2})} \times \frac{2}{(\cancel{1-x^2})}$$

$$f'(x) = 2\ln\left(\frac{1+x}{1-x}\right) - 2$$

$$(a^2)^3 = a^{3\times 2}$$

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$$f(x) = e^{-\frac{x^2}{2}} \cdot |x|^x$$

$$\left(e^{\ln x}\right)^2 = (x)^2$$

$$e^{2\ln x} = x^2$$

$$e^{3\ln x} = x^3$$

$$e^{n\ln x} = x^n$$

$$\left(e^{\ln x}\right)^x = (x)^x$$

$$\boxed{e^{x\ln x} = x^x}$$

On a : $e^{x\ln|x|} = |x|^x$

$$\Rightarrow f(x) = e^{-\frac{x^2}{2}} \cdot e^{x\ln|x|}$$

$$= e^{x\ln|x| - \frac{x^2}{2}}$$

On pose $u(x) = x\ln|x| - \frac{x^2}{2}$

$$(e^u)' = u' e^u$$

$$f(x) = e^{u(x)}$$

$$\Rightarrow f'(x) = \left(x\ln|x| - \frac{x^2}{2}\right)' e^{x\ln|x| - \frac{x^2}{2}}$$

$$= \left(\ln|x| + x \times \frac{1}{x} - x\right) e^{x\ln|x| - \frac{x^2}{2}}$$

$$(\ln|u|)' = \frac{u'}{u}$$

$$= (\ln|x| + 1 - x)\, e^{x\ln|x| - \frac{x^2}{2}}$$

$$(\ln|x|)' = \frac{1}{x}$$